# MINIMIZATION TECHNIQUES

→ Binary Logic is used in all of today's digital Computers and devices, the cost of the circuits that implement it is an important factor.

→ Finding simpler and cheaper, but equivalent, realizations of a circuit must be good. To reducing the overall cost of the design.

→

## Boolean theorems :-

1. Complementation Laws :-

Complement means, to change 0's to 1s and 1s to 0's.

Law 1 = $\overline{0} = 1$

Law 2 = $\overline{1} = 0$

Law 3 = $A = 0 \rightarrow \overline{A} = 1$

Law 4 = $A = 1 \rightarrow \overline{A} = 0$.

Law 5 = $\overline{\overline{A}} = A$ (Double complementation does not change the function).

2. AND laws :-

Law 1 : $A \cdot 0 = 0$

Law 2 = $A \cdot 1 = A$

Law 3 = $A \cdot A = A$

Law 4 = $A \cdot \overline{A} = 0$.

3. OR laws :-

Law 1 = $A + 1 = 1$

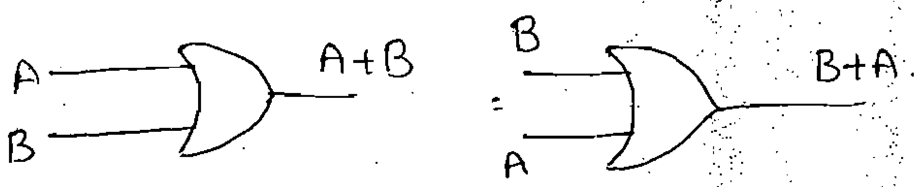
Law 2 = $A + 0 = A$

Law 3 : $A + A = A$

Law 4 : $A + \overline{A} = 1$

4. Commutative laws :-

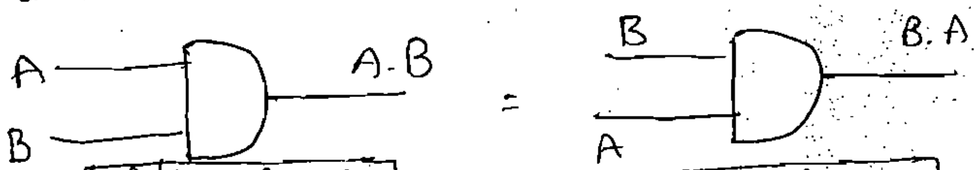
Law 1 :- $A + B = B + A$




| A | B | A+B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

| A | B | B+A |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Law2 :- $A \cdot B = B \cdot A$.

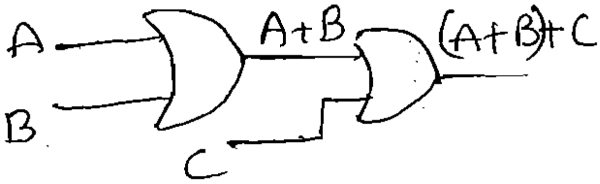


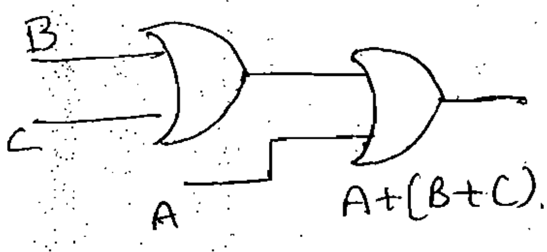

| A | B | A.B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| A | B | B.A |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

5. Associate laws :-

Law 1: $(A+B)+C = A+(B+C)$

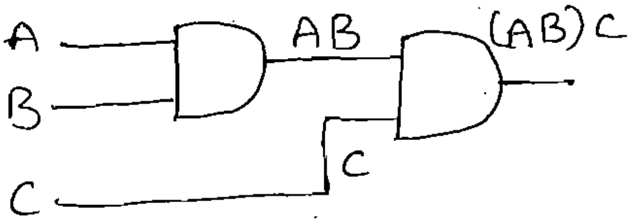


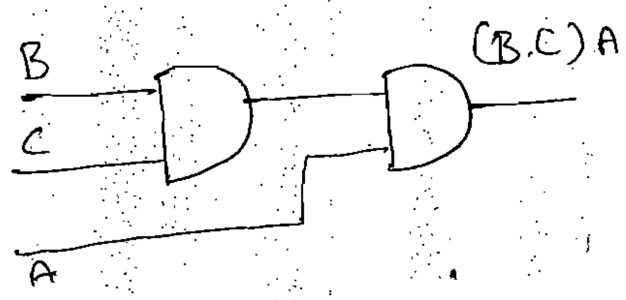



| A | B | C | A+B | (A+B)+C |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| A | B | C | B+C | A+(B+C) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

law 2: $(A.B)C = A(B.C)$.



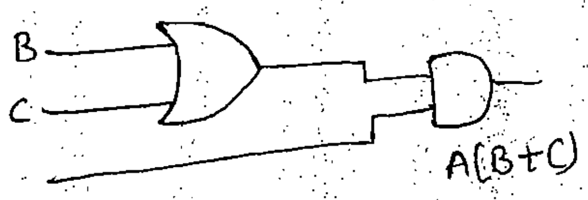




| A | B | C | AB | (AB) C |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

| A | B | C | B.C | A(B.C) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

6. Distributive laws :-

Law 1 : $A(B+C) = AB + AC$

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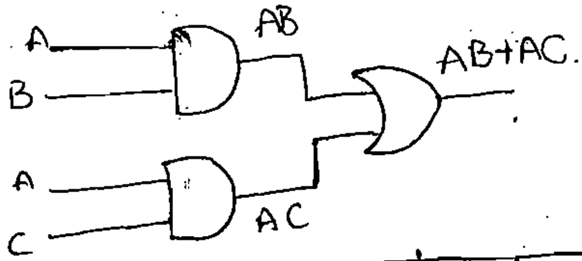
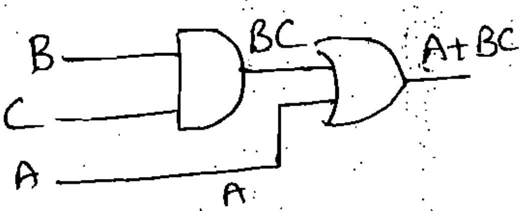
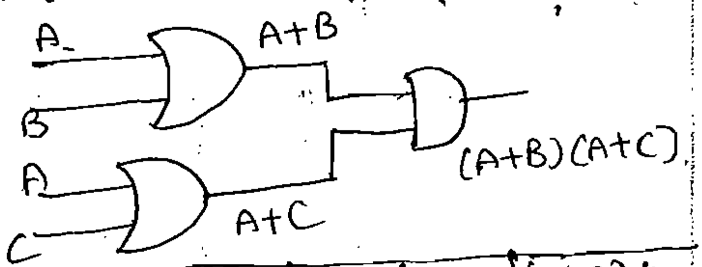



| A | B | C | (B+C) | A(B+C) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| A | B | C | AB | AC | AB+AC |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

Law 2 : $A + BC = (A+B)(A+C)$



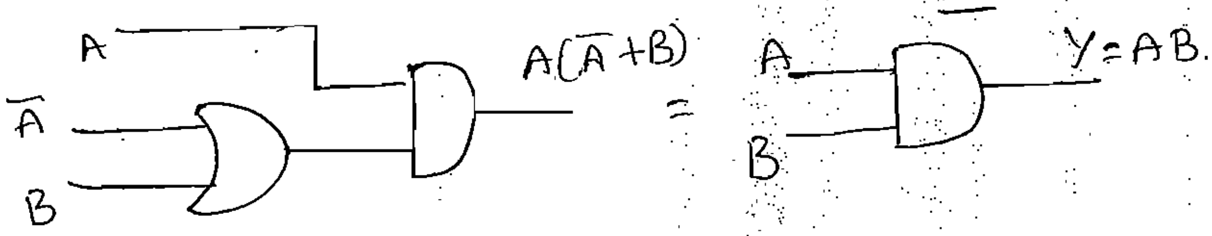

| A | B | C | BC | A+BC |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| A | B | C | A+B | A+C | (A+B)(A+C |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

7. Redundant literal Rule :-

Law 1: $A(\bar{A}+B) = AB.$

$$= A(\bar{A}+B)$$
$$= A.\bar{A} + AB$$
$$= \underline{AB}$$




| A | B | $\bar{A}+B$ | $A(\bar{A}+B)$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

| A | B | A.B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Law2: $A+\bar{A}B. = A+B.$

$$= A+\bar{A}B$$
$$= (A+\bar{A})(A+B)$$
$$= (A+B)$$

| A | B | $\bar{A}B$ | $A+\bar{A}B$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 |

| A | B | A+B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

8. Idempotence laws :-

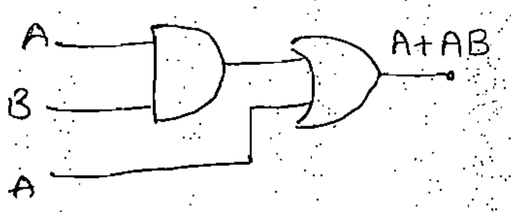
Idempotence means the same value.

Law 1 = $A \cdot A = A$ — if $A=0$ then $0.0=0$; if $A=1$ then $1 \cdot 1 = 1$

Law = $A+A = A$ — if $A=0$ then $0+0=0$; if $A=1$ then $1+1=1$

9. Absorption laws :-

Law 1 = $A + A \cdot B = A$




| A | B | A·B | A+A·B |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |

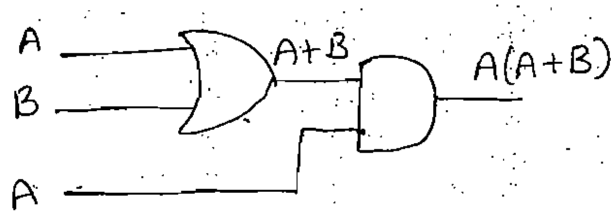
$= A + A \cdot B$

$= A(1+B)$ $\because 1+B=1$

$= A \cdot 1$

$= A.$

Law 2 : $A(A+B) = A$




| A | B | A+B | A(A+B) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

$= A \cdot A + AB$

$= A + AB$

$= A(1+B)$

$= A.$

$\therefore$ $= A(A + \text{any term})$

$= A$

10. Consensus Theorem :-

Theorem 1 : $AB + \overline{A}C + BC = AB + \overline{A}C.$

L.H.S $\rightarrow$ $AB + \overline{A}C + BC$

$= AB + \overline{A}C + BC(A + \overline{A})$

$= AB + \overline{A}C + ABC + \overline{A}BC$

$= AB(1+C) + \overline{A}C(1+B)$

$= AB(1) + \overline{A}C = AB + \overline{A}C.$

$$AB+\overline{A}C+BCD = AB+\overline{A}C$$

L.H.S $\rightarrow AB+\overline{A}C+BCD$

$AB+\overline{A}C+BCD(A+\overline{A})$

$AB+\overline{A}C+ABCD+\overline{A}BCD$

$AB(1+CD)+\overline{A}C(1+CD)$

$AB+\overline{A}C$.

Theorem 2 : $(A+B)(\overline{A}+C)(B+C) = (A+B)(\overline{A}+C)$.

L.H.S : $(A+B)(\overline{A}+C)(B+C)$

$(A\overline{A}+AC+\overline{A}B+BC)(B+C)$

$(AC+BC+\overline{A}B)(B+C)$

$ABC+BC+\overline{A}BC+AC+BC+\overline{A}B$.

$= BC+AC+\overline{A}B(1+C)+ABC$

$= BC+\overline{A}B+AC(1+B)$

$= AC+BC+\overline{A}B$.

R.H.S $= (A+B)(\overline{A}+C)$

$= A\cdot\overline{A}+AC+\overline{A}B+BC$

$= \overline{A}B+AC+BC$.

$\rightarrow (A+B)(\overline{A}+C)(B+C+D) = (A+B)(\overline{A}+C)$.

If a sum of products comprises a term containing A and a term containing $\overline{A}$, and a third term containing the left out literals of the first two terms, then the third term is redundant.

$\rightarrow$ The function remains the same with or without the third term removed or retained.

## Transposition Theorem :-

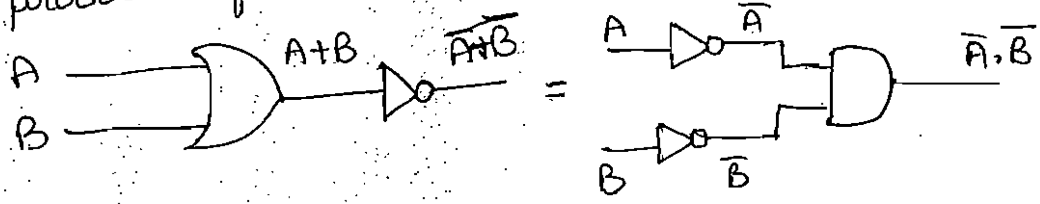
Theorem : $AB+\bar{A}C = (A+C)(\bar{A}+B)$

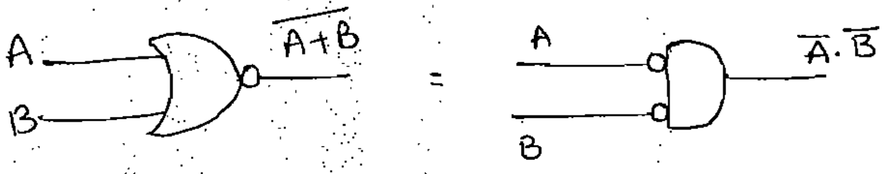
$$
\begin{aligned}
R.H.S &= (A+C)(\bar{A}+B) \\
&= A.\bar{A} + AB + \bar{A}C + BC \\
&= AB + \bar{A}C + BC \\
&= AB + \bar{A}C + BC(A+\bar{A}) \\
&= AB + \bar{A}C + ABC + \bar{A}BC \\
&= AB(1+C) + \bar{A}C(1+B) \\
&= AB + \bar{A}C
\end{aligned}
$$

## Demorgan's Theorem :-

Law I $\quad \overline{A+B} = \bar{A}.\bar{B}$

The complement of a sum of variables is equal to the product of their individual complements.




| A | B | A+B | $\overline{A+B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

| A | B | $\bar{A}$ | $\bar{B}$ | $\bar{A}.\bar{B}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |

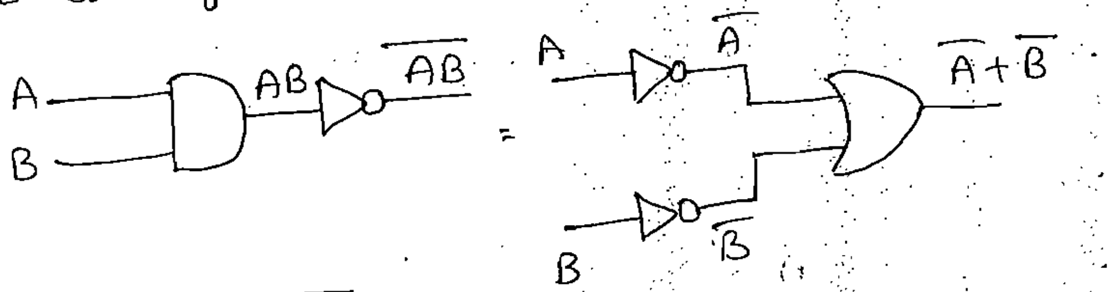
Similarly $\rightarrow$ $\overline{A+B+C+D+E} = \overline{A}.\overline{B}.\overline{C}.\overline{D}.\overline{E}$

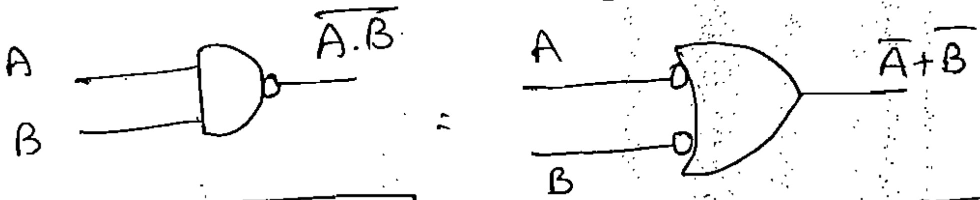
$\rightarrow$ $\overline{\overline{A}B+C\overline{D}+\overline{E}\overline{D}} = (\overline{\overline{A}B})(\overline{C\overline{D}})(\overline{\overline{E}\overline{D}})$

$= (A+\overline{B})(\overline{C}+D)(E+D)$

Law 2 $\quad \overline{AB} = \overline{A}+\overline{B}$

The complement of the product of variables is equal to the sum of their individual complements.




| A | B | A.B | $\overline{A.B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

| A | B | $\overline{A}$ | $\overline{B}$ | $\overline{A}+\overline{B}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |

Duality:-

when changing from one logic system to another, '0' becomes '1' and '1' becomes '0'. An AND gate becomes an OR gate and an OR gate becomes an AND gate. Given a Boolean identity, we can produce a dual identity by changing all '+' sign to '.' signs, all '.' signs to '+' signs. The variables are not complemented.

$\rightarrow$ $A.(A\cdot B) = A\cdot B$ $\quad\rightarrow$ $A+A+B = A+B.$

$\rightarrow$ $\overline{A}B+\overline{A}+AB = 0$ $\quad\rightarrow$ $\overline{A}+B\cdot\overline{A}\cdot(A+B) = 1$

$\rightarrow$ $A+B = AB+\overline{A}B+A\overline{B}$ $\quad\rightarrow$ $AB = (A+B)(\overline{A}+B)(A+\overline{B})$

## Reducing Boolean expressions by using Boolean theorems :-

* $f = A[B+\bar{C}(\overline{AB+A\bar{C}})]$

$f = A[B+\bar{C}((\overline{AB})\cdot(\overline{A\bar{C}}))]$ (or)

$= A[B+\bar{C}(\bar{A}+\bar{B})(\bar{A}+\bar{\bar{C}})]$

$= A[B+\bar{C}[(\bar{A}+\bar{B})(\bar{A}+C)]]$

$= A[B+\bar{C}[\bar{A}\bar{A}+\bar{A}C+\bar{A}\bar{B}+\bar{B}C]]$ $\qquad \bar{A}\cdot\bar{A} = \bar{A}$

$= A[B+\bar{C}[\bar{A}+\bar{A}C+\bar{A}\bar{B}+\bar{B}C]]$

$= A[B+\bar{A}\bar{C}+\bar{A}C\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{B}C\bar{C}]$ $\qquad C\cdot\bar{C} = 0$

$= A[B+\bar{A}\bar{C}+\bar{A}\bar{B}\bar{C}]$

$= AB+A\bar{A}\bar{C}+A\bar{A}\bar{B}\bar{C}$ $\qquad A\cdot\bar{A} = 0.$

$= AB$

* $f = (\bar{A}+B)(\bar{B}+C)+(AB+C)$

$= \bar{A}\bar{B}+\bar{A}C+B\bar{B}+BC+AB+C$ $\qquad B\cdot\bar{B} = 0$

$= \bar{A}\bar{B}+\bar{A}C+BC+AB+C$

$= C(1+\bar{A}+B)+AB+\bar{A}\bar{B}$

$= AB+\bar{A}\bar{B}+C.$

* $f = (\overline{A+B})(\overline{ABC})+(\overline{\bar{A}C})$

$= (\bar{A}\cdot\bar{B})(\bar{A}+\bar{B}+\bar{C})+A+\bar{C}$

$= \bar{A}\bar{A}+\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{A}\bar{B}+\bar{B}\bar{B}+\bar{B}\bar{C}+A+\bar{C}$

$= \bar{A}+\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{B}+\bar{B}\bar{C}+A+\bar{C}$

$= \bar{A}(1+\bar{B}+\bar{C}) + \bar{B}(1+\bar{C}) + A+\bar{C}$

$= \bar{A}+\bar{B}+A+\bar{C} = A+\bar{A}+\bar{B}+\bar{C} = 1+\bar{B}+\bar{C} = \underline{1}$

* $f = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= \underbrace{AA}_{A}+A\bar{B}+A\bar{C}+AB+\underbrace{\bar{B}B}_{0}+B\bar{C}+A\bar{C}+\bar{B}\bar{C}+\underbrace{\bar{C}\bar{C}}_{0}\;(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= A+A\bar{B}+A\bar{C}+AB+B\bar{C}+\bar{B}\bar{C}\;(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= \underbrace{\bar{A}BA}_{0}+\underbrace{A\bar{B}\bar{A}}_{0}+\underbrace{A\bar{C}\bar{A}}_{0}+\underbrace{AB\bar{A}}_{0}+B\bar{C}\bar{A}+\bar{B}\bar{C}\bar{A}+AB+\underbrace{A\bar{B}B}_{0}+\underbrace{A\bar{C}B}_{B\bar{C}}+\underbrace{ABB}_{AB}$
$\underbrace{B\cdot B\bar{C}}_{B\bar{C}}+\underbrace{\bar{B}\bar{C}B}_{0}+A\bar{C}+A\bar{B}\bar{C}+\underbrace{A\bar{C}\bar{C}}_{A\bar{C}}+AB\bar{C}+\underbrace{B\bar{C}\bar{C}}_{B\bar{C}}+\underbrace{\bar{B}\bar{C}\bar{C}}_{\bar{B}\bar{C}}\;(\bar{A}+\bar{B}+\bar{C})$

$= \bar{A}B\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+AB+AB\bar{C}+AB+B\bar{C}+A\bar{C}+A\bar{B}\bar{C}+A\bar{C}+AB\bar{C}$
$+B\bar{C}+\bar{B}\bar{C}(\bar{A}+\bar{B}+\bar{C})$

$= \bar{A}B\bar{C}+\bar{A}B\bar{C}+\bar{A}\bar{B}\bar{C}+\overset{0}{AB\bar{A}}+AB\bar{C}\bar{A}+\overset{0}{AB\bar{A}}+B\bar{C}\bar{A}+\overset{0}{A\bar{C}\bar{A}}+\overset{0}{A\bar{B}\bar{C}\bar{A}}+\overset{0}{A\bar{C}\bar{A}}$
$+\overset{0}{A\bar{A}BC}+B\bar{C}\bar{A}+\bar{B}\bar{C}\bar{A}+\overset{0}{\bar{A}BC\bar{B}}+\overset{0}{\bar{A}BC\bar{B}}+\bar{A}\bar{B}\bar{C}\bar{B}+A\bar{B}\bar{B}+A\bar{B}\bar{C}\bar{B}+$
$\overset{0}{AB\bar{B}}+\overset{0}{B\bar{C}\bar{B}}+A\bar{B}\bar{C}+A\bar{B}\bar{C}\bar{B}+A\bar{C}\bar{B}+A\bar{B}\bar{C}\bar{B}+B\bar{C}\bar{B}+\bar{B}\bar{C}\bar{B}+\bar{A}\bar{B}\bar{C}$
$+\bar{A}B\bar{C}+\bar{A}\bar{B}\bar{C}+AB\bar{C}+AB\bar{C}+AB\bar{C}+B\bar{C}\bar{C}+A\bar{C}\bar{C}+A\bar{B}\bar{C}\bar{C}+A\bar{C}\bar{C}$
$+A\bar{B}\bar{C}\bar{C}+B\bar{C}\bar{C}+\bar{B}\bar{C}\bar{C}.$

$= \bar{A}B\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}\bar{C}+\bar{B}\bar{C}+AB\bar{C}+B\bar{C}+A\bar{C}$

$= \bar{B}\bar{C}(1+\bar{A}+A)+\bar{A}B\bar{C}+AB\bar{C}+B\bar{C}+A\bar{C}$

$= \bar{B}\bar{C}+\bar{A}B\bar{C}+AB\bar{C}+B\bar{C}+A\bar{C}$

$= B\bar{C}(1+A)+\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}\bar{C}$

$= B\bar{C}+\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{C}$

$= B\bar{C}(1+\bar{A})+\bar{B}\bar{C}+A\bar{C}$

$= B\bar{C}+\bar{B}\bar{C}+A\bar{C}$

$= \bar{C}(B+\bar{B}+A) = \bar{C}(1+A) = \bar{C}$

$$* \; f = A\cdot\left[\overline{\left(\overline{A\oplus B}\right)\oplus C}\right]$$

$$\rightarrow f = A\cdot\left[\overline{\underbrace{\left(\overline{A\oplus B}\right)}_{A}\oplus\underbrace{C}_{B}}\right]$$

$$= A\left[\overline{\overline{\overline{A\oplus B}}\cdot C + (\overline{A\oplus B})\cdot\overline{C}}\right]$$

$$= A\left[\overline{(\overline{A}B+\overline{B}A)\cdot C}\cdot\left(\overline{(\overline{\overline{A}B+\overline{B}A})\cdot\overline{C}}\right)\right]$$

$$= A\left[(\overline{\overline{A}B})(\overline{\overline{B}A})+\overline{C})\cdot(\overline{\overline{\overline{A}B+\overline{B}A}}+\overline{\overline{C}})\right]$$

$$= A\left[(\overline{\overline{A}}+\overline{B})(\overline{\overline{B}}+\overline{A})+\overline{C}\right]\cdot(\overline{A}B+\overline{B}A+C)]$$

$$= A\left[(AB+A\overline{A}+B\overline{B}+\overline{A}\,\overline{B}+\overline{C})(\overline{A}B+\overline{B}A+C)\right]$$

$$= A\left[AB+\overline{A}\,\overline{B}+\overline{C})(\overline{A}B+\overline{B}A+C)\right]$$

$$= A\left[AB\overline{A}B+AB\overline{B}A+ABC+\overline{A}\,\overline{B}\,\overline{A}B+\overline{A}\,\overline{B}\,\overline{B}A+\overline{A}\,\overline{B}C+\overline{A}B\overline{C}+\overline{B}A\overline{C} + C\cdot\overline{C}\right]$$

$$= A\left[ABC+\overline{A}\,\overline{B}C+\overline{A}B\overline{C}+A\overline{B}\,\overline{C}\right]$$

$$= A\cdot ABC + A\cdot\overline{A}\,\overline{B}C + A\cdot\overline{A}B\overline{C} + A\cdot A\overline{B}\,\overline{C}$$

$$= ABC + A\overline{B}\,\overline{C}$$

$$= A\left[BC+\overline{B}\,\overline{C}\right]$$

$$= A\left[B\odot C\right].$$

$$* \; f = (B+BC)(B+\overline{B}C)(B+D).$$

$$f = (B\cdot B + B\overline{B}C + BC\cdot B + BC\overline{B}C)(B+D)$$

$$= (B+BC)(B+D)$$

$$= B\cdot B + BD + B\cdot BC + BCD$$

$$= B + BD + BC + BCD$$

$$= B(1+C) + BD(1+C)$$

$$= B + BD \Rightarrow B(1+D) = \underline{B}$$

* $f(A,B,C,D) = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{D} + CD$

Applying the consensus theorem to 2nd and 4th terms

$$\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{D} + CD + \bar{B}D \qquad (\bar{B}\bar{C} + CD + \bar{B}D = \bar{B}\bar{C} + CD)$$

3rd and 5th terms, the term $\bar{A}\bar{B}$ becomes redundant.

$$\bar{B}\bar{C} + \bar{A}\bar{D} + CD \qquad (\bar{B}D + \bar{A}\bar{D} + \bar{A}\bar{B} = \bar{B}D + \bar{A}\bar{D})$$

$$f_{min} = \bar{A}\bar{D} + \bar{B}\bar{C} + CD$$

K-map (Karnaugh-map) :-

The K-map method, on the other hand, is a systematic method of (simplification dep) simplifying the Boolean Expressions. The K-map is a chart or a graph, composed of an arrangement of adjacent cells, each representing a particular combination of variables in sum or product form.

→ An n variable function can have $2^n$ possible combinations of product terms in SOP form, or $2^n$ possible combinations of sum terms in POS form.

→ A Two-variable K-map will have $2^2 = 4$ cells or squares.

→ a three-variable K-map will have $2^3 = 8$ cells or squares

→ a four-variable K-map will have $2^4 = 16$ cells or squares.

→ Any boolean Expression can be expressed in a standard or expanded sum of products form or in a standard or canonical or expanded product of sums form.

→ Each of the product terms in the standard SOP form is called minterms and each of the sum terms in the standard POS form is called maxterms.

Two-Variable K-map :- (Mapping of SOP Expressions).

A two-variable K-map has $2^2 = 4$ cells or squares. 4 possible combinations of the input variables A and B. ($\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$, $AB$).

$m_0 = \bar{A}\bar{B}$, $m_1 = \bar{A}B$, $m_2 = A\bar{B}$, $m_3 = AB$.

* $f = A\bar{B} + AB$ simplify by using K-map.

| A \ B | 0 | 1 |
|---|---|---|
| 0 | (0) | (1) |
| 1 | 1 (2) | 1 (3) |

$f = B$.

| A \ B | | |
|---|---|---|
| | $\bar{A}\bar{B}$ | $\bar{A}B$ |
| | $A\bar{B}$ | $AB$ |

The minterms of a two-variable K-map.

* Reduce the expression by using K-map.

$f = A\bar{B} + AB + \bar{A}B$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | | 1 |
| 1 | 1 | 1 |

$f = A+B$

logic diagram


mapping of POS Expressions :-

$M_0 = A+B$, $M_1 = A+\bar{B}$, $M_2 = \bar{A}+B$, $M_3 = \bar{A}+\bar{B}$

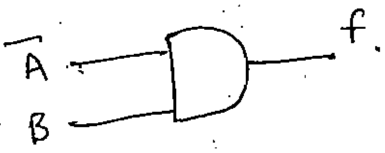
| A \ B | 0 | 1 |
|---|---|---|
| 0 | $A+B$ (0) | $A+\bar{B}$ (1) |
| 1 | $\bar{A}+B$ (2) | $\bar{A}+\bar{B}$ (3) |

The maxterms of a two-variable K-map.

→ Reduce the expression by using K-map

$$f = (A+B)(\bar{A}+\bar{B})(\bar{A}+B)$$

| A \ B | 0 | 1 |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | 0 |




$$f = \bar{A}.B$$

**Three- variable K-map :-**

A function in three variable (A,B,C) Expressed in the standard SOP form can have $2^3 = 8$ possible combinations they are $\bar{A}\bar{B}\bar{C}$, $\bar{A}\bar{B}C$, $\bar{A}B\bar{C}$, $A\bar{B}\bar{C}$, $\bar{A}BC$, $A\bar{B}\bar{C}$, $AB\bar{C}$, and ABC. in the standard pos form can have $2^3 = 8$ possible combinations. They are $A+B+C$, $A+B+\bar{C}$, $A+\bar{B}+C$, $A+\bar{B}+\bar{C}$, $\bar{A}+B+C$, $\bar{A}+B+\bar{C}$, $\bar{A}+\bar{B}+C$ and $\bar{A}+\bar{B}+\bar{C}$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $\bar{A}\bar{B}\bar{C}$ $(m_0)_0$ | $\bar{A}\bar{B}C$ $(m_1)_1$ | $\bar{A}BC$ $(m_3)_3$ | $\bar{A}B\bar{C}$ $(m_2)_2$ |
| 1 | $A\bar{B}\bar{C}$ $(m_4)_4$ | $A\bar{B}C$ $(m_5)_5$ | $ABC$ $(m_7)_7$ | $AB\bar{C}$ $(m_6)_6$ |

minterms

| | | | |
|---|---|---|---|
| 0 $A+B+C$ $(M_0)$ | 1 $A+B+\bar{C}$ $(M_1)$ | 3 $A+\bar{B}+\bar{C}$ $(M_3)$ | 2 $A+\bar{B}+C$ $(M_2)$ |
| 4 $\bar{A}+B+C$ $(M_4)$ | 5 $\bar{A}+B+\bar{C}$ $(M_5)$ | 7 $\bar{A}+\bar{B}+\bar{C}$ $(M_7)$ | 6 $\bar{A}+\bar{B}+C$ $(M_6)$ |

max terms.

* Reduce the expression $f = \bar{A}\bar{B}C + A\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$.

$\bar{A}\bar{B}C = 001 = m_1$,

$A\bar{B}C = 101 = m_5$

$\bar{A}B\bar{C} = 010 = m_2$

$AB\bar{C} = 110 = m_6$.

$ABC = 111 = m_7$

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 (1) | 3 | 1 (2) |
| 1 | 4 | 1 (5) | 1 (7) | 1 (6) |

$f_1 = m_1 + m_5 = \bar{A}\bar{B}C + A\bar{B}C = \bar{B}C(A+\bar{A}) = \bar{B}C.$

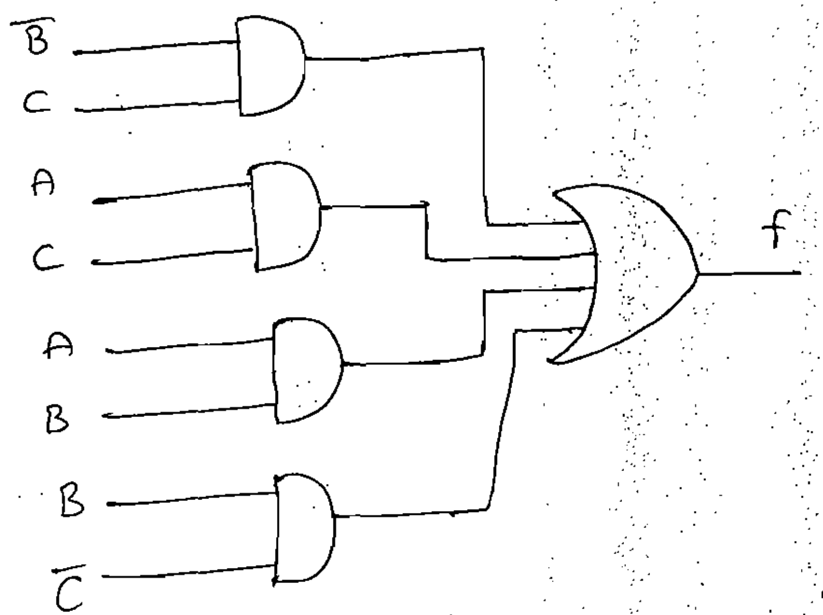
$f_2 = m_5 + m_7 = A\bar{B}C + ABC = AC(B+\bar{B}) = AC$

$f_3 = m_7 + m_6 = ABC + AB\bar{C} = AB(C+\bar{C}) = AB$

$f_4 = m_2 + m_6 = AB\bar{C} + \bar{A}B\bar{C} = B\bar{C}(A+\bar{A}) = B\bar{C}$

$f = f_1 + f_2 + f_3 + f_4$

$= \bar{B}C + AC + AB + B\bar{C}$




logic diagram

* Reduce the Expression. $f = (A+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$
$(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

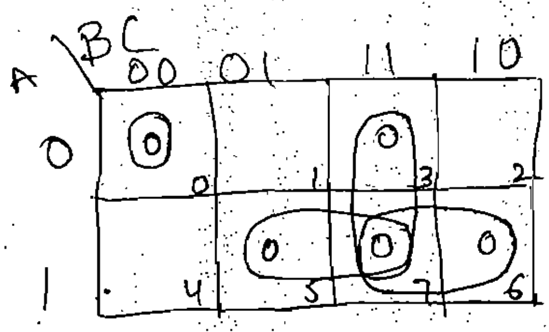
$A+B+C = 000 = M_0$

$\bar{A}+B+\bar{C} = 101 = M_5$

$\bar{A}+\bar{B}+\bar{C} = \phi 11 = M_7$

$A+\bar{B}+\bar{C} = 011 = M_3.$

$\bar{A}+\bar{B}+C = 110 = M_6$



$f_1 = M_0 = A+B+C$

$f_2 = M_5 \cdot M_7 = [\bar{A}+B+\bar{C}][\bar{A}+\bar{B}+\bar{C}]$

$= \bar{A}\bar{A}+\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{A}B+\bar{B}B+B\bar{C}+\bar{A}\bar{C}+\bar{B}\bar{C}+\bar{C}\bar{C}$

$= \bar{A}\bar{A}+\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{A}B+\bar{B}B+B\bar{C}+\bar{A}\bar{C}+\bar{B}\bar{C}+\bar{C}\bar{C}$

$= \bar{A}+\bar{A}\bar{B}+\bar{A}\bar{C}+\bar{A}B+B\bar{C}+\bar{B}\bar{C}$

$= \bar{A}(1+\bar{B}+\bar{C}+B)+\bar{C}(B+\bar{B})$

$f_2 = \bar{A}+\bar{C}$

$f_3 = M_3 \cdot M_7$

$= (A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= A\bar{A}+A\bar{B}+A\bar{C}+\bar{A}\bar{B}+\bar{B}\bar{B}+\bar{B}\bar{C}+\bar{A}\bar{C}+\bar{B}\bar{C}+\bar{C}\bar{C}$

$= A\bar{B}+A\bar{C}+\bar{A}\bar{B}+\bar{B}+\bar{B}\bar{C}+\bar{A}\bar{C}+\bar{C}$

$= \bar{B}(1+\bar{A}+A+\bar{C})+\bar{C}(1+\bar{B}+\bar{A})$

$= \bar{B}+\bar{C}$

$f_4 = M_7 \cdot M_6$

$= (\bar{A}+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$

$= \bar{A}\cdot\bar{A}+\bar{A}\bar{B}+\bar{A}C+\bar{A}\bar{B}+\bar{B}\bar{B}+\bar{B}C+\bar{A}\bar{C}+\bar{B}\bar{C}+C\cdot\bar{C}$

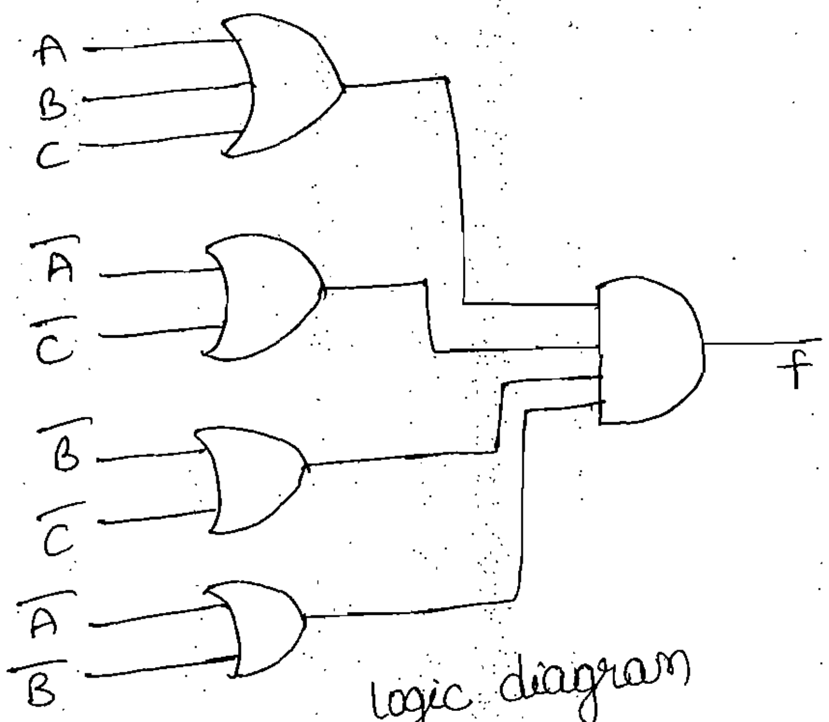
$= \bar{A}+\bar{A}\bar{B}+\bar{A}C+\bar{B}+\bar{B}C+\bar{A}\bar{C}+\bar{B}\bar{C}+0$

$= \bar{A}(1+\bar{B}+C+\bar{C})+\bar{B}(1+C+\bar{C})$

$= \bar{A}+\bar{B}$

$F = f_1 \cdot f_2 \cdot f_3 \cdot f_4$

$= (A+B+C)(\bar{A}+\bar{C})(\bar{B}+\bar{C})(\bar{A}+\bar{B})$




logic diagram

## Four-variable K-map :-

A four-variable (A,B,C,D) expression can have $2^4 = 16$ possible combinations.

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $\bar{A}\bar{B}\bar{C}\bar{D}$ 0 | $\bar{A}\bar{B}\bar{C}D$ 1 | $\bar{A}\bar{B}CD$ 3 | $\bar{A}\bar{B}C\bar{D}$ 2 |
| 01 | $\bar{A}B\bar{C}\bar{D}$ 4 | $\bar{A}B\bar{C}D$ 5 | $\bar{A}BCD$ 7 | $\bar{A}BC\bar{D}$ 6 |
| 11 | $AB\bar{C}\bar{D}$ 12 | $AB\bar{C}D$ 13 | $ABCD$ 15 | $ABC\bar{D}$ 14 |
| 10 | $A\bar{B}\bar{C}\bar{D}$ 8 | $A\bar{B}\bar{C}D$ 9 | $A\bar{B}CD$ 11 | $A\bar{B}C\bar{D}$ 10 |

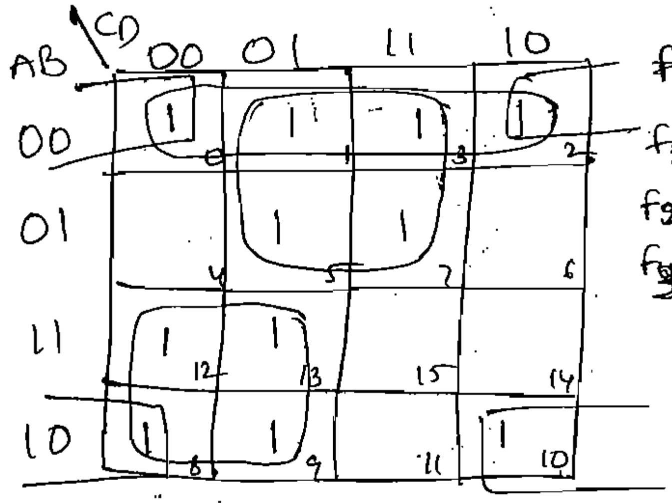
Sop form.

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $A+B+C+D$ 0 | $A+B+C+\bar{D}$ 1 | $A+B+\bar{C}+\bar{D}$ 3 | $A+B+\bar{C}+D$ 2 |
| 01 | $A+\bar{B}+C+D$ 4 | $A+\bar{B}+C+\bar{D}$ 5 | $A+\bar{B}+\bar{C}+\bar{D}$ 7 | $A+\bar{B}+\bar{C}+D$ 6 |
| 11 | $\bar{A}+\bar{B}+C+D$ 12 | $\bar{A}+\bar{B}+C+\bar{D}$ 13 | $\bar{A}+\bar{B}+\bar{C}+\bar{D}$ 15 | $\bar{A}+\bar{B}+\bar{C}+D$ 14 |
| 10 | $\bar{A}+B+C+D$ 8 | $\bar{A}+B+C+\bar{D}$ 9 | $\bar{A}+B+\bar{C}+\bar{D}$ 11 | $\bar{A}+B+\bar{C}+D$ 10 |

pos form.

* Reduce the expression $f = \Sigma m(0,1,2,3,5,7,8,9,10,12,13)$ and implement the expression by using universal logic.

$f = \Sigma m(0,1,2,3,5,7,8,9,10,12,13)$




$f_1 = \cancel{m_0+m_1+m_2+m_3} = \cancel{\overline{A}\overline{B}}$

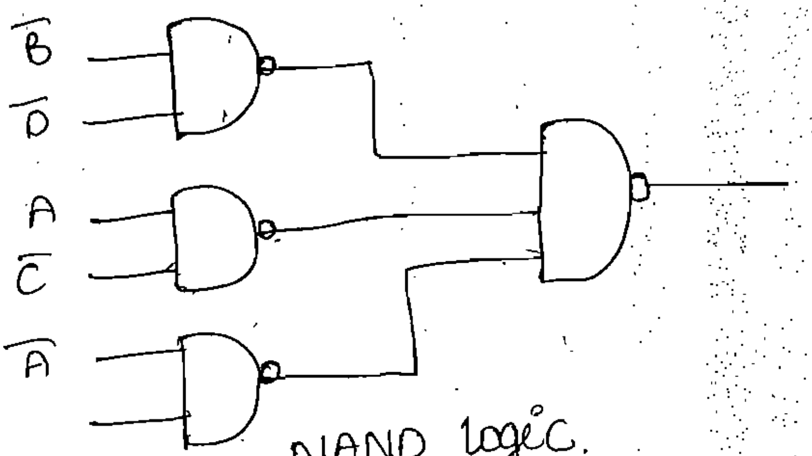
$f_1 = m_1+m_3+m_5+m_7 = \overline{A}D$

$f_2 = m_{12}+m_{13}+m_8+m_9 = A\overline{C}$

$f_3 = m_8+m_{10}+m_0+m_2 = \overline{B}\overline{D}$

$f_{min} = f_1+f_2+f_3$

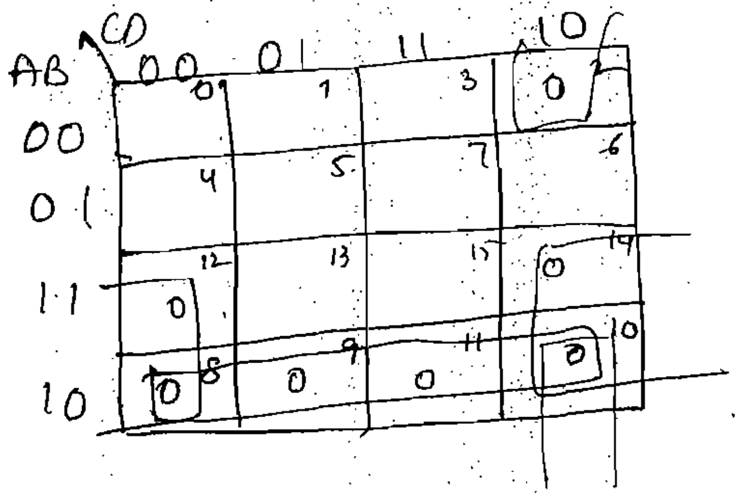
$= \overline{B}\overline{D} + A\overline{C} + \overline{A}D$




NAND logic.

* Reduce the expression $f = \Pi M(2,8,9,10,11,12,14)$ and implement the expression by using universal logic.

$f = \Pi M(2,8,9,10,11,12,14)$.



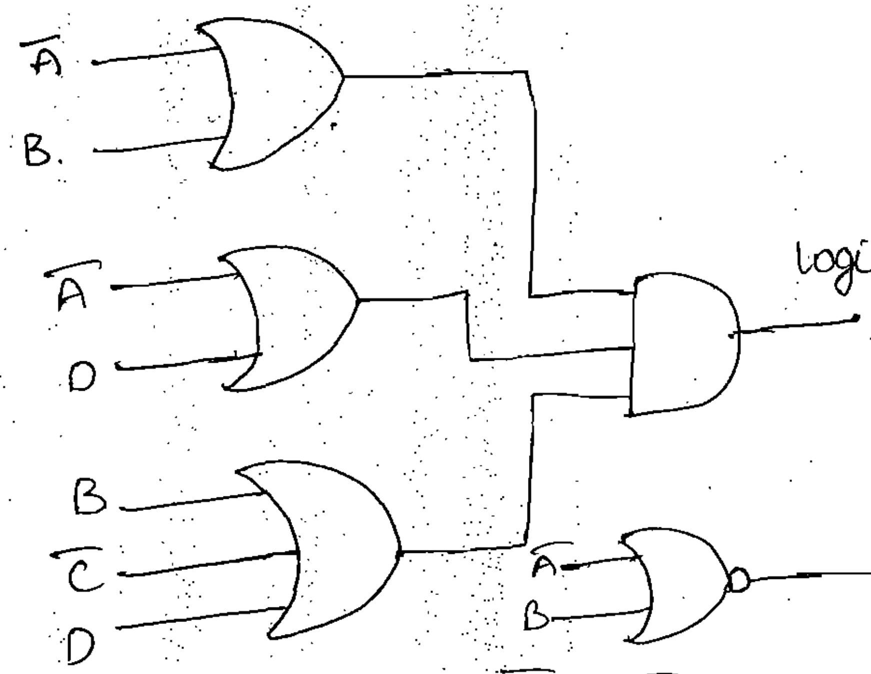

$f_1 = M_8 \cdot M_9 \cdot M_{11} \cdot M_{10}$ $\quad$ $f_2 = M_8 \cdot M_{12} \cdot M_{10} \cdot M_{14}$

$= (\bar{A}+B)$ $\quad$ $= (\bar{A}+D)$.

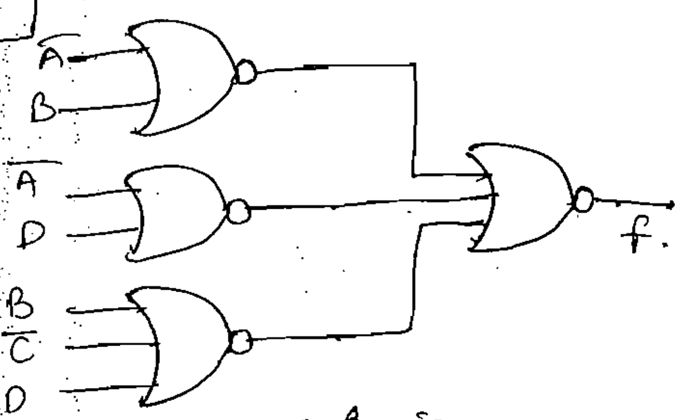
$f_3 = M_2 \cdot M_{10}$

$= (B+\bar{C}+D)$.

$f_{min} = (\bar{A}+B)(\bar{A}+D)(B+\bar{C}+D)$




logic diagram.




NOR logic.

prime implicants :- [PI]

Each square or rectangular made up of the bunch of adjacent minterms is called a subcube. Each of these subcubes is called a prime implicants.

essential prime implicants:-

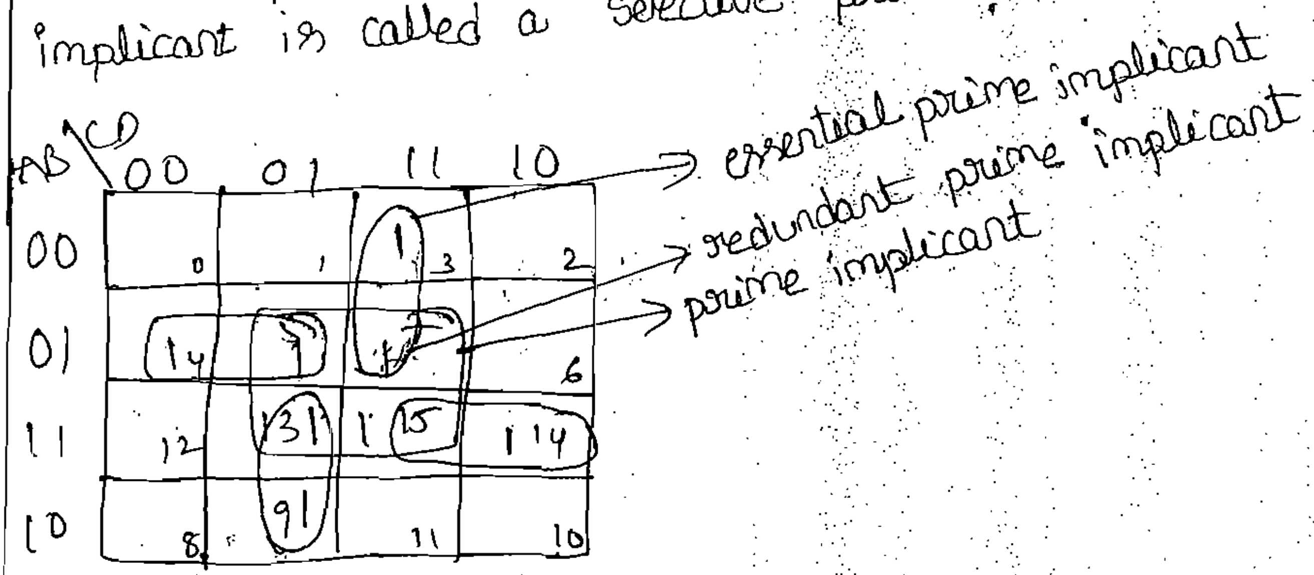
The prime implicants which contains at least one 1 which cannot be covered by any other prime implicant is called an essential prime implicant [EPI]

Redundant prime implicants:-

The prime implicant whose each 1 is covered at least by one EPI is called a redundant prime implicants. (RPI).

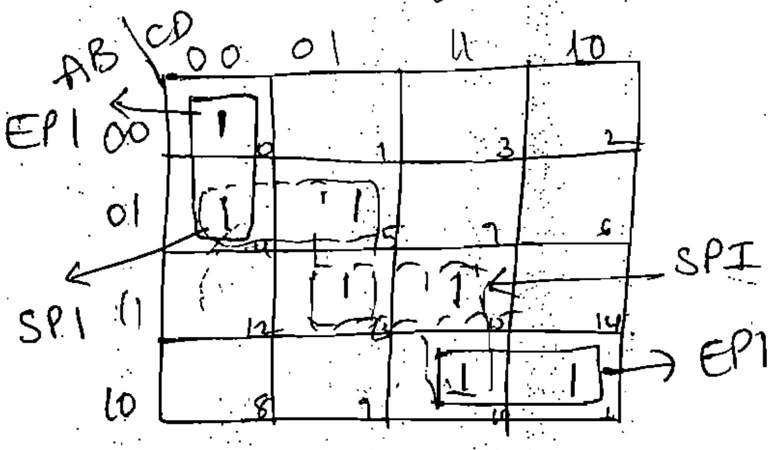
Selective prime implicants :-

A prime implicant which is neither an essential prime implicant nor a redundant prime implicant is called a selective prime implicant (SPI).




$f = \Sigma m(5,7,13,15,9,14,4)$.

F(A,B,C,D) = $\sum m(0,4,5,10,11,13,15)$.

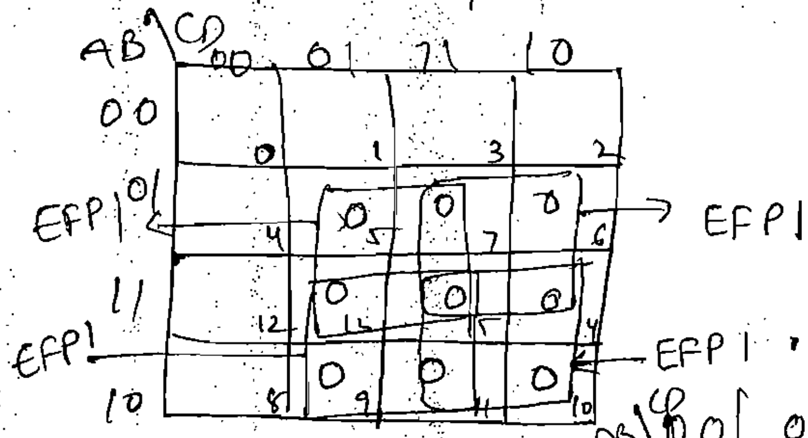


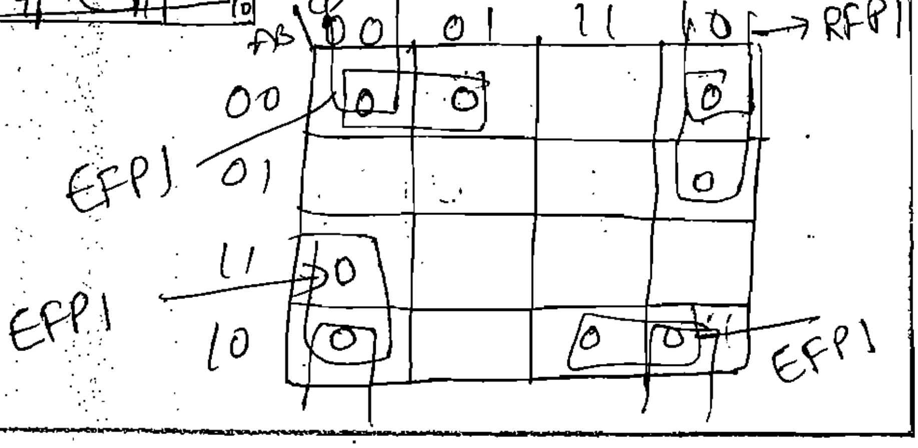

False prime implicants :- (FPI)

The prime implicants obtained by using the maxterms are called False prime implicants.

Essential false prime implicants :-

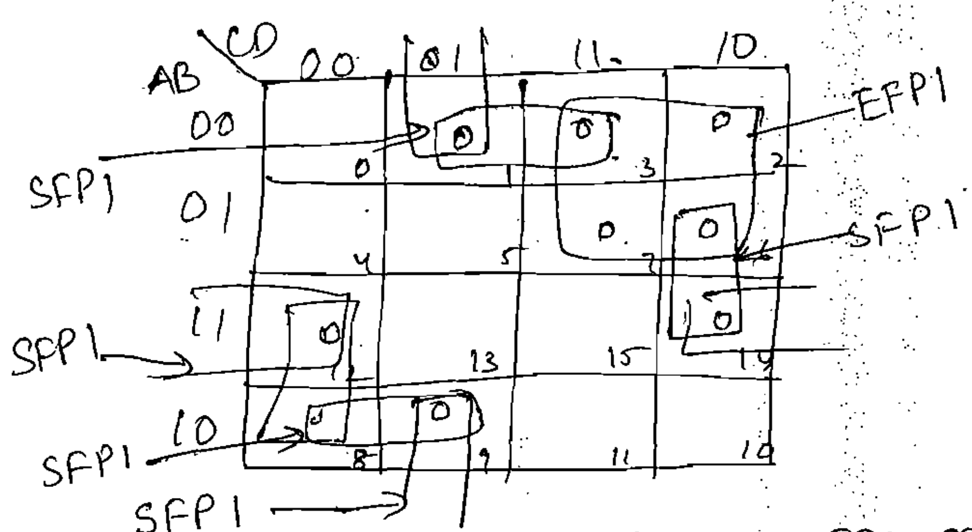
The FPI's which contains at least one '0' which cannot be covered by any other FPI is called Essential False prime implicants (EFPI).



The four corner 0's from the largest cluster of adjacent 0's, which is an FPI whose 0's are covered by essential FPIs and hence is a redundant false prime implicant (RFPI)

$$F(A,B,C,D) = (A+\bar{C})(A+B+\bar{D})(\bar{A}+B+C)(\bar{A}+\bar{B}+D)$$




The function has in all seven FPIs marked in figure.

The FPI is an essential. FPI as it contains 0s at locations 2 and 7 which cannot be covered by any other FPI.

The remaing FPI are all SFPIs.

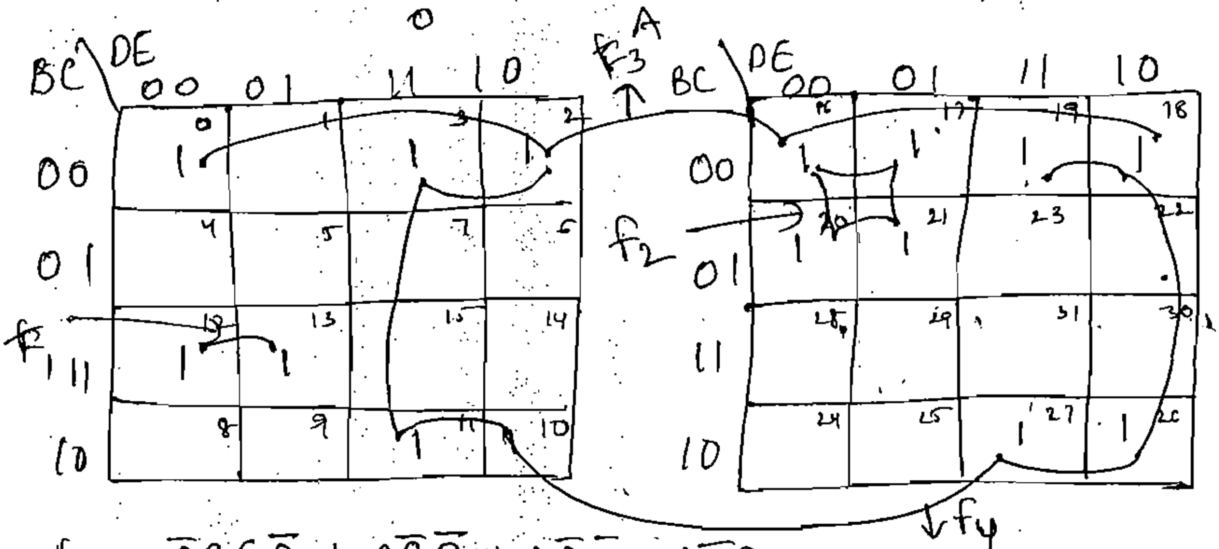
## Five-variable K-map :-

A five- variable (A,B,C,D,E) expression can have $2^5 = 32$ possible combinations of input variables.

The 32 squares of the K-map are divided into 2 blocks of 16 squares each. one block taken as a A='0'. and another one taken as a A='1'.

* Reduce the following expression in SOP and POS form.

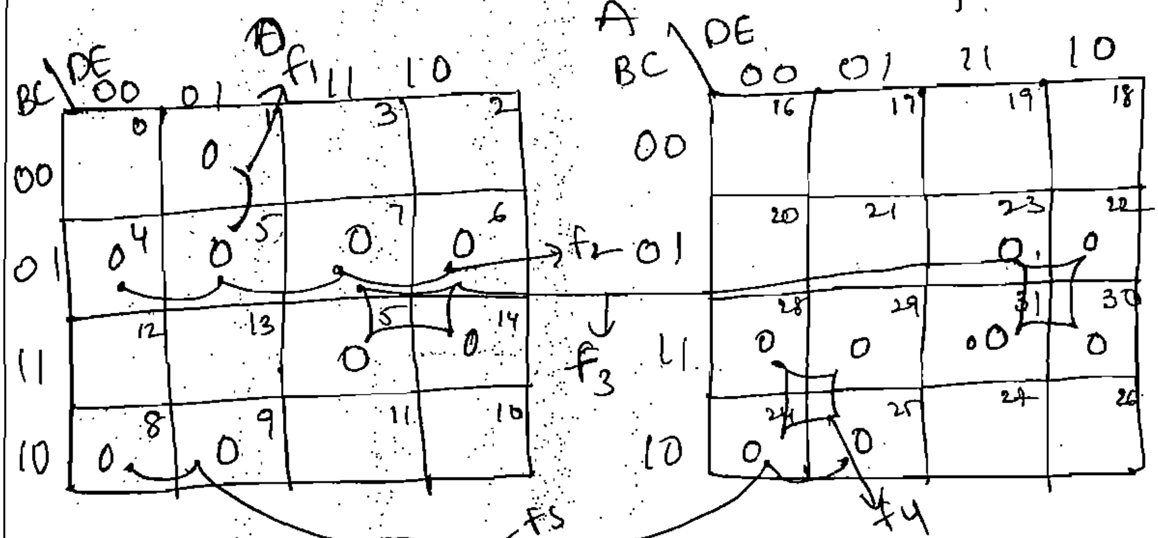
$$f = \sum m(0,2,3,10,11,12,13,16,17,18,19,20,21,26,27)$$



$$f_{min} = \overline{A}BC\overline{D} + A\overline{B}\overline{D} + A\overline{B}\overline{D} + A\overline{C}D$$

$\qquad f_1 \qquad f_2 \qquad f_3 \qquad f_4.$

$$f = \prod M(1,4,5,6,7,8,9,14,15,22,23,24,25,28,29,30,31)$$



$f_1 = (A+B+D+\overline{E})$ $\qquad f_3 = \overline{C}+\overline{D}$ $\qquad f_5 = (\overline{B}+C+D)$

$f_2 = (A+B+\overline{C})$ $\qquad f_4 = \overline{A}+\overline{B}+D$

$$F_{min} = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5$$

$$= (A+B+D+\overline{E})(A+B+\overline{C})(\overline{C}+\overline{D})(\overline{A}+\overline{B}+D)(\overline{B}+C+D)$$

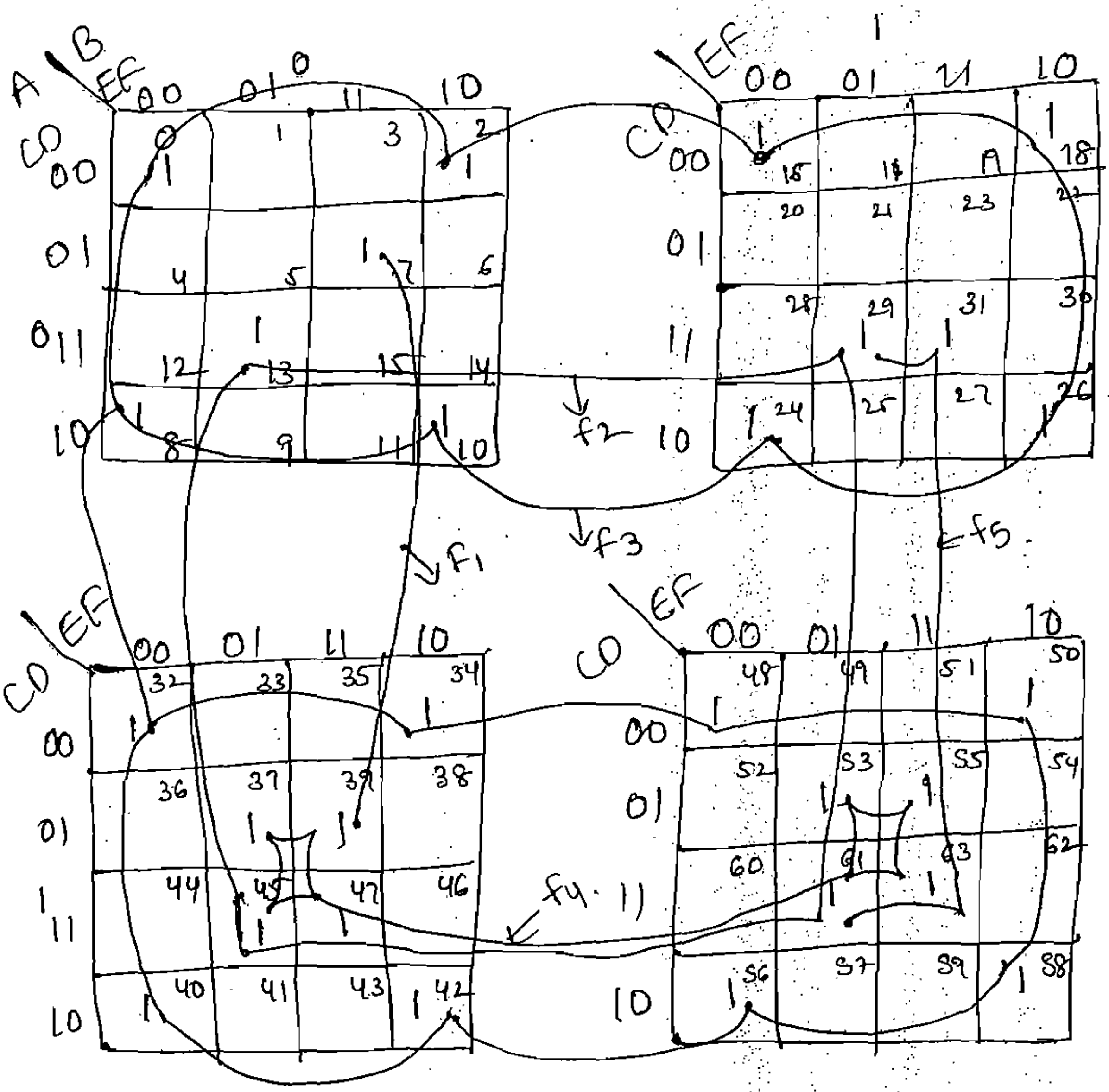
## Six - Variable K-map :-

A Six- variable (A,B,C,D,E,F) expression can have $2^6 = 64$ possible combinations of input variables.

The 64 squares of the K-map are divided into 4 blocks of 16 squares each. one block taken as a AB = "00" and remaing are "01", "10", and "11".

* Reduce the expression

$f = \Sigma m(0,2,7,8,10,13,16,18,24,26,29,31,32,34,37,39,40,42,45,$
$47,48,50,53,55,56,58,61,63)$.



$f_1 = \overline{B}\overline{C}DEF$

$f_2 = CD\overline{E}F$

$f_3 = \overline{D}\overline{F}$

$f_4 = ADF$

$f_5 = BCDF$

$$F_{min} = f_1 + f_2 + f_3 + f_4 + f_5$$

$$F_{min} = \overline{B}\overline{C}DEF + CD\overline{E}\overline{F} + \overline{D}\overline{F} + ADF + BCDF.$$

Don't care combinations :-

The combinations for which the values of the expression are not specified are called don't care combinations or optional combinations and such expressions, therefore in completely specified. The don't care terms are denoted by d, X or $\phi$. During the process of design using an sop map. each don't care is treated as a 1 if it is helpful in map reduction. During the process of design using an POS map each don't care is treated as a 0 if it is helpful in map reduction.

→ A standard sop expression with don't cares can be converted into a standard pos form by keeping the don't cares as they are, and writing the missing minterms of the sop form as the maxterms of the pos form.

→ A standard pos expression with don't cares can be converted into a standard sop form by keeping the don't cares as they are and writing the missing maxterms of the pos form as the min terms of the sop form.

$F(w,x,y,z) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,5).$

| wx \ yz | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | d (0) | 1 (1) | 1 (3) | d (2) |
| 01 | (4) | d (5) | 1 (7) | (6) |
| 11 | (12) | (13) | 1 (15) | (14) |
| 10 | (8) | (9) | 1 (11) | (10) |

$\rightarrow f_1 = \overline{w}\,\overline{x}$

$\rightarrow f_2 = yz$

$F_{min} = f_1 + f_2$

$= \overline{w}\overline{x} + yz$.

$F(A,B,C,D) = \Pi M(0,3,7,8,9,10,11,15) . \Pi d(2,4).$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 (0) | (1) | 0 (3) | d (2) |
| 01 | d (4) | (5) | 0 (7) | (6) |
| 11 | (12) | (13) | 0 (15) | (14) |
| 10 | 0 (8) | 0 (9) | 0 (11) | 0 (10) |

$\rightarrow f_1$

$\rightarrow f_2$

$\rightarrow f_3$

$f_1 = \overline{C} + \overline{D}$

$f_2 = \overline{A} + B$

$f_3 = (B + D)$.

$F_{min} = f_1 f_2 f_3 = (\overline{C} + \overline{D})(\overline{A} + B)(B + D)$.

Limitations of K-map :-

→ K-maps are not suitable when the number of variables involved exceed four. It may be used with difficulty up to five and six variable systems. But beyond 'six variable' K-maps cannot be physically visualized.

→ The K-map simplification is a manual technique and simplification process is heavily dependent on the abilities of the designer. It cannot be programmed.

→ Reduce the expression $f = \sum m(1,5,6,12,13,14) + d(2,4)$ by using k-map in POS form and implement by using universal logic

$f = \sum m(1,5,6,12,13,14) + d(2,4) \rightarrow$ SOP form.

$f = \prod M(0,3,7,8,9,10,11,15) \cdot \prod d(2,4)$.

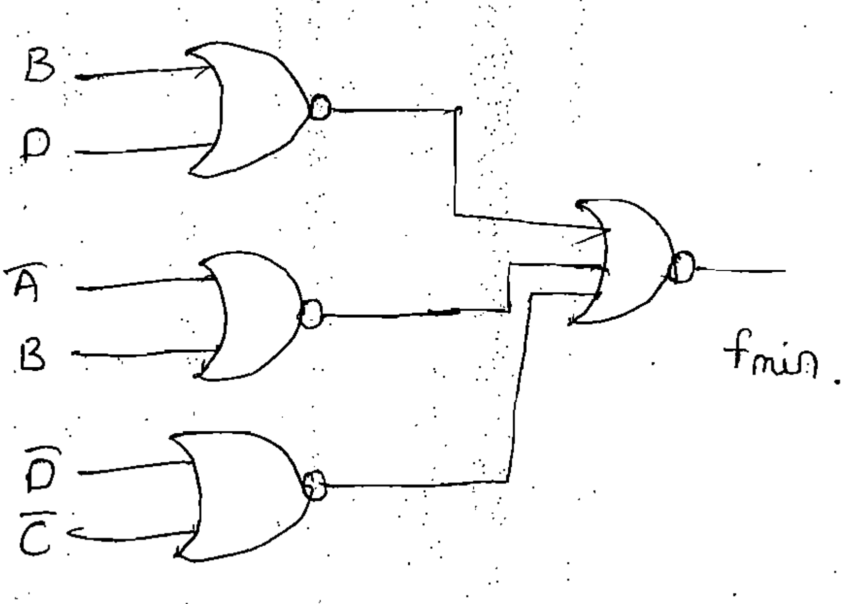
| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 (0) | (1) | 0 (3) | d (2) |
| 01 | d (4) | (5) | 0 (7) | (6) |
| 11 | (12) | (13) | 0 (15) | (14) |
| 10 | 0 (8) | 0 (9) | 0 (11) | 0 (10) |

$f_1 = \overline{C} + \overline{D}$

$f_2 = (B + D)$

$f_3 = (\overline{A} + B)$

$f_{min} = (\overline{C} + \overline{D})(B + D)(\overline{A} + B)$.



Quine McCluskey or tabular method :-

W.V. Quine and E.J. McCluskey developed an exact tabular method to simplify the Boolean Expression. This methode is called the Quine McCluskey or tabular method.

The procedure for the minimization of a Boolean expression by the tabular methode

Step 1. list all the minterms.

Step 2 : arrange all minterms in groups of the same number of 1s in their binary representation in column 1.

Step 3. compare each term of the lowest index group with every term in the succeeding group.

Step 4 :- compare the terms generated in step2 in the same fashion combine two terms which differ by only a single 1 and whose dashes are in the same position to generate a new term.

Step 5 :- list all the prime implicants and draw the implicant chart. (The don't cares if any should not apper in the prime implicant chart).

Step 6 :- obtain essential prime implicants and write the minimal expression.

* obtain the minimal expression for $f = \Sigma m(1,2,3,5,6,7,8,9,12,13,15)$ using tabular methode.

**Step 1**

| | | |
|---|---|---|
| index 1 | 1 | 0001 ✓ |
| | 2 | 0010 ✓ |
| | 8 | 1000 ✓ |
| index 2 | 3 | 0011 ✓ |
| | 5 | 0101 ✓ |
| | 6 | 0110 ✓ |
| | 9 | 1001 ✓ |
| | 12 | 1100 ✓ |
| index 3 | 7 | 0111 ✓ |
| | 13 | 1101 ✓ |
| index 4 | 15 | 1111 ✓ |

**Step 2**

| | | |
|---|---|---|
| ✓ (1,3) | (2) | 00–1 |
| ✓ (1,5) | (4) | 0–01 |
| ✓ (1,9) | (8) | –001 |
| ✓ (2,3) | (1) | 001– |
| ✓ (2,6) | (4) | 0–10 |
| ✓ (8,9) | (1) | 100– |
| ✓ (8,12) | (4) | 1–00 |
| ✓ (3,7) | (4) | 0–11 |
| ✓ (5,7) | (2) | 01–1 |
| ✓ (5,13) | (8) | –101 |
| ✓ (6,7) | (1) | 011– |
| ✓ (9,13) | (4) | 1–01 |
| ✓ (12,13) | (1) | 110– |
| (7,15) | (8) | –111 |
| (13,15) | (2) | 11–1 |

**Step 3**

| | | | |
|---|---|---|---|
| (1,3,5,7) | (2,4) | 0– –1 | T |
| (1,5,9,13) | (4,8) | – –01 | S |
| (2,3,6,7) | (1,4) | 0–1– | R |
| (8,9,12,13) | (1,4) | 1–0– | Q |
| (5,7,13,15) | (2,8) | –1–1 | P |

**Prime implicant chart**

| | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 12 | 13 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| * P → 5,7,13,15 (2,8) | | | | X | | X | | | | X | X |
| * Q → 8,9,12,13 | | | | | | | X | X | X | X | |
| * R → 2,3,6,7 | | X | X | | X | X | | | | | |
| S → 1,5,9,13 | X | | | X | | | | X | | X | |
| T → 1,3,5,7 | X | | X | X | | X | | | | | |

$P + Q + R + S \rightarrow BD + A\bar{C} + \bar{A}C + \bar{C}D$

$P + Q + R + T \rightarrow BD + A\bar{C} + \bar{A}C + \bar{A}D$.

Simplify the following function using the branching method

$f(A,B,C,D,E) = \Sigma m(0,4,12,16,19,24,28,29,31)$

| | | |
|---|---|---|
| Index 0 | 0 | 00000 ✓ |
| index 1 | 4 | 00100 ✓ |
| | 16 | 10000 ✓ |
| index 2 | 12 | 01100 ✓ |
| | 24 | 11000 ✓ |
| index 3 | 19 | 10011 X |
| | 28 | 11100 ✓ |
| index 4 | 29 | 11101 ✓ |
| index 5 | 31 | 11111 ✓ |

| | | | |
|---|---|---|---|
| (0,4) | (4) | 00-00 | W ✓ |
| (0,16) | (16) | -0000 | V |
| (4,12) | (8) | 0-100 | U |
| (16,24) | (8) | 1-000 | T ✓ |
| (12,28) | (16) | -1100 | S ✓ |
| (24,28) | (4) | 11-00 | R ✓ |
| (28,29) | (1) | 1110- | Q |
| (29,31) | (2) | 111-1 | P * |

Prime implicant chart

| | 0 | 4 | 12 | 16 | 19 | 24 | 28 | 29 | 31 |
|---|---|---|---|---|---|---|---|---|---|
| * P (29,31) | | | | | | | | X | X |
| Q (28,29) | | | | | | | X | X | |
| R (24,28) | | | | | | X | X | | |
| ✓ S (12,28) | | | X | | | | X | | |
| ✓ T (16,24) | | | | X | | X | | | |
| U (4,12) | | X | X | | | | | | |
| V (0,16) | X | | | X | | | | | |
| ✓ W (0,4) | X | X | | | | | | | |
| * X (19) | | | | | X | | | | |

In table X and P are essential prime implicants.

| | 0 | 4 | 12 | 16 | 24 | 28 |
|---|---|---|---|---|---|---|
| w(0,4) (4) | X | X | | | | |
| V(0,16) (16) | X | | | X | | |
| U(4,12) (8) | | X | X | | | |
| T(16,24) (8) | | | | X | X | |
| S(12,28) (16) | | | X | | | X |
| R(24,28) (4) | | | | | X | X |
| Q(28,29) (1) | | | | | | X |

In the reduced PI chart, there are no essential PIs, dominated rows or dominating columns. In column 0, it is covered by rows w and V. First select row w.

| | 12 | 16 | 24 | 28 |
|---|---|---|---|---|
| V(0,16) | | X | | |
| U(4,12) | X | | | |
| T(16,24) | | X | X | |
| S(12,28) | X | | | X |
| R(24,28) | | | X | X |
| Q(28,29) | | | | X |

In this row V is dominated by row T, row U and row Q are dominated by row S. So rows V, U, Q can be reduced.

| | 12 | 16 | 24 | 28 |
|---|---|---|---|---|
| * T(16,24) | | X | X | |
| * S(12,28) | X | | | X |
| R(24,28) | | | X | X |

In this T and S are the secondary essential prime implicants. So R is Redundant.

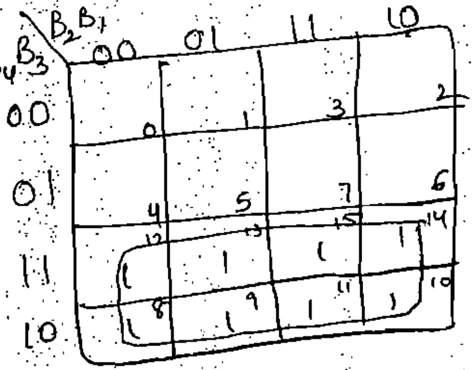
$$f_{min} = w + S + T + X + P$$
$$= \bar{A}\bar{B}\bar{D}\bar{E} + BC\bar{D}\bar{E} + A\bar{C}\bar{D}\bar{E} + A\bar{B}\bar{C}DE + ABCE$$

## Code converters :-

Design a circuit 4-bit binary to Gray code converters.

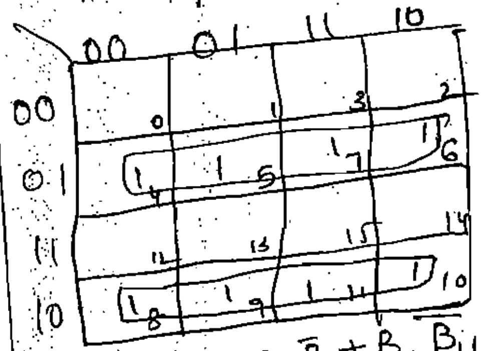
| 4-bit binary | | | | 4-bit Gray | | | |
|---|---|---|---|---|---|---|---|
| $B_4$ | $B_3$ | $B_2$ | $B_1$ | $G_4$ | $G_3$ | $G_2$ | $G_1$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

K-map for $G_4$



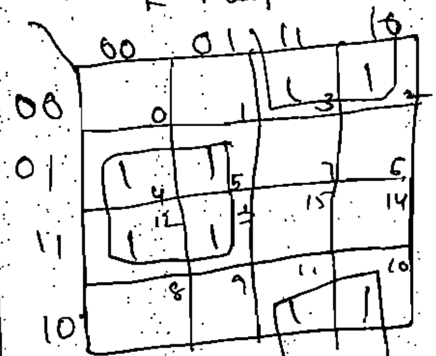
$G_4 = B_4$

K-map for $G_3$



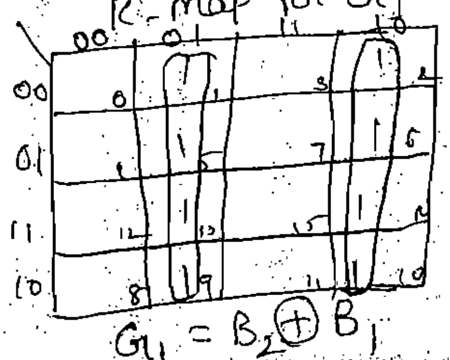
$G_3 = B_4\bar{B}_3 + B_3\bar{B}_4$
$= B_3 \oplus B_4$

K-map for $G_2$

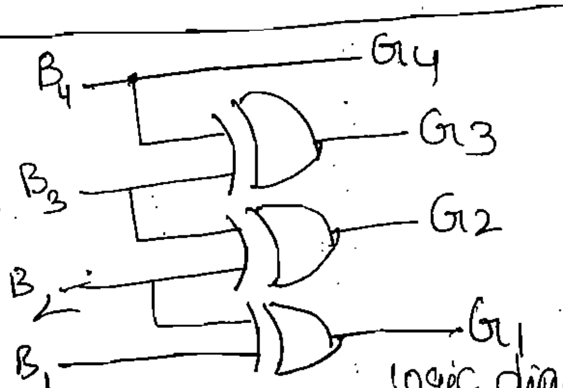


$G_2 = B_3 \oplus B_2$

K-map for $G_1$



$G_1 = B_2 \oplus B_1$

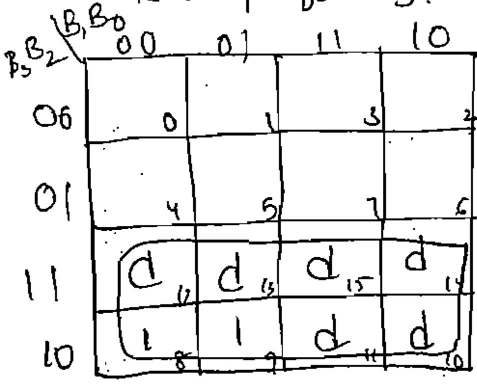



logic diagram

## Design of a BCD to Gray Code converter :-

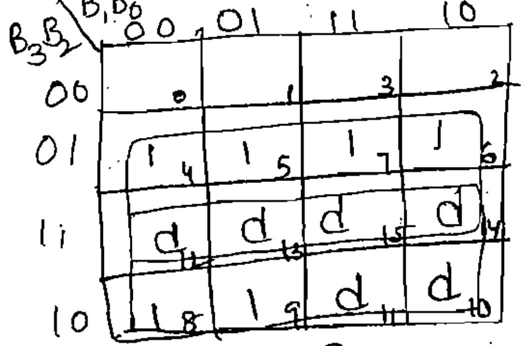
| BCD Code | | | | Gray Code | | | |
|---|---|---|---|---|---|---|---|
| $B_3$ | $B_2$ | $B_1$ | $B_0$ | $G_3$ | $G_2$ | $G_1$ | $G_0$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

K-map for $G_3$.




$G_3 = B_3$.

K-map for $G_2$




$G_2 = B_2 \oplus B_3$.

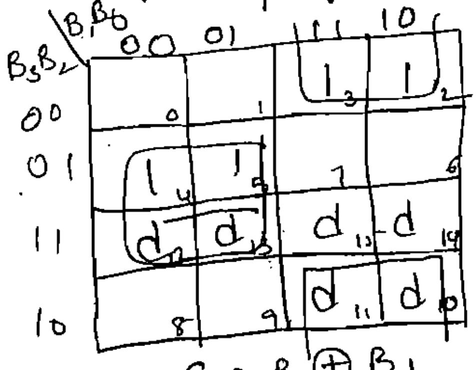
$G_3 = \Sigma m(8,9) + d(10,11,12,13,14,15)$

$G_2 = \Sigma m(4,5,6,7,8,9) + d(10,11,12,13,14,15)$

$G_1 = \Sigma m(2,3,4,5) + d(10,11,12,13,14,15)$

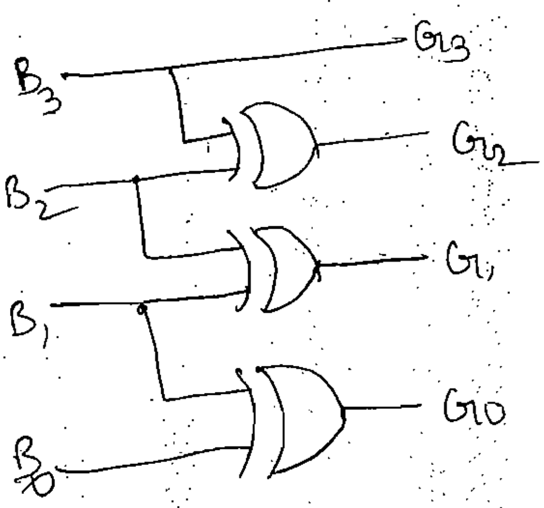
$G_0 = \Sigma m(1,2,5,6,9) + d(10,11,12,13,14,15)$

K-map for $G_1$

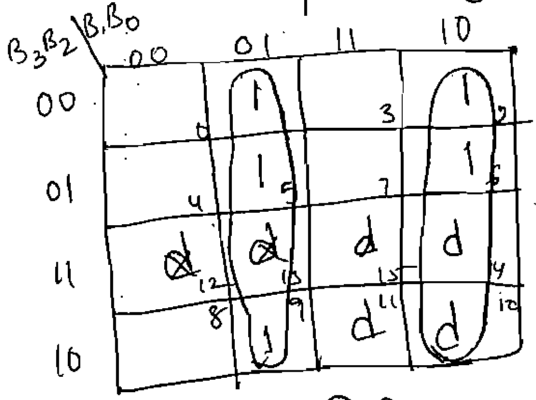



$G_1 = B_2 \oplus B_1$.

logic diagram.




K-map for $G_0$



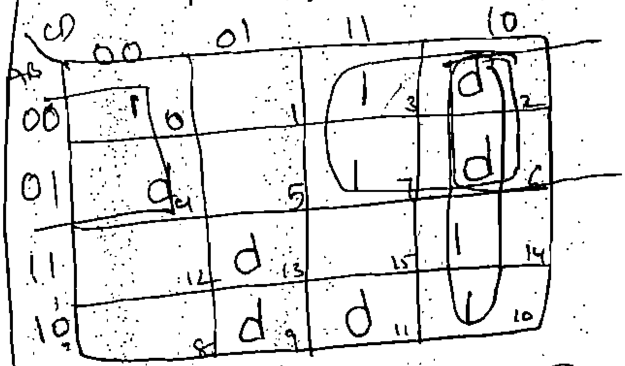

$G_0 = B_1 \oplus B_0$.

→ Design an SOP circuit to detect the decimal numbers 0, 2, 4, 6 and 8 in a 4-bit 5211 BCD code input.

| Decimal number | 5211 Code A B C D | output f |
|---|---|---|
| 0 | 0 0 0 0 | 1 |
| 1 | 0 0 0 1 | 0 |
| 2 | 0 0 1 1 | 1 |
| 3 | 0 1 0 1 | 0 |
| 4 | 0 1 1 1 | 1 |
| 5 | 1 0 0 0 | 0 |
| 6 | 1 0 0 1 | 1 |
| 7 | 1 0 1 1 | 0 |
| 8 x | 1 1 1 0 | 1 |
| 9 | 1 1 1 1 | 0 |

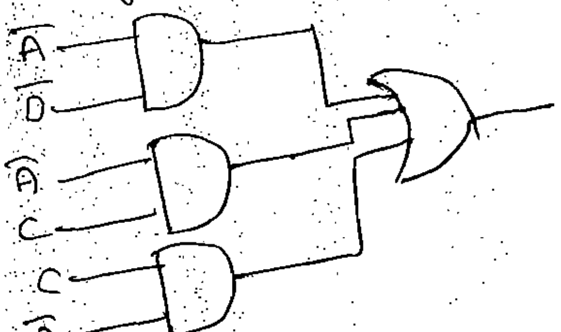
$f = \Sigma m(0,3,7,10,14) + d(2,4,6,9,11,13)$

K-map



$f_{min} = \bar{A}\bar{D} + \bar{A}C + C\bar{D}$

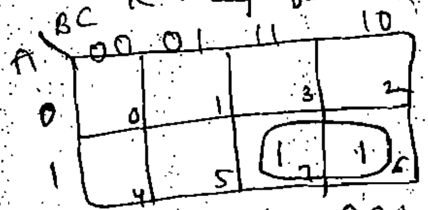
logic diagram



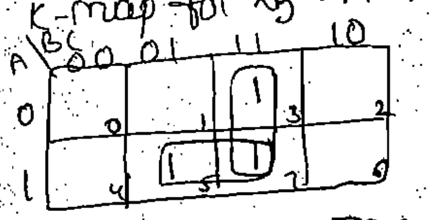
→ Design a combinational circuit that accepts a 3-bit BCD number and generates an output binary number equal to the square of the input number.

| Inputs A | B | C | outputs $X_6$ | $X_5$ | $X_4$ | $X_3$ | $X_2$ | $X_1$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

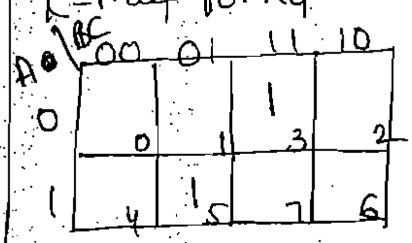
K-map for $X_6 = AB$



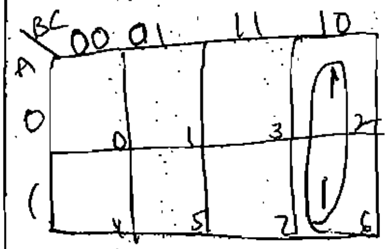
K-map for $X_5 = AC + BC$



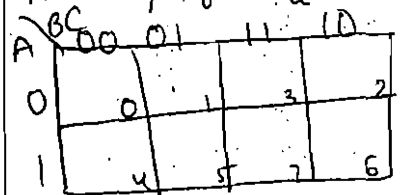
K-map for $X_4 = \bar{A}BC + A\bar{B}C$



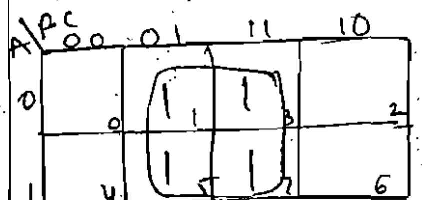
K-map for $X_3 = B\overline{C}$

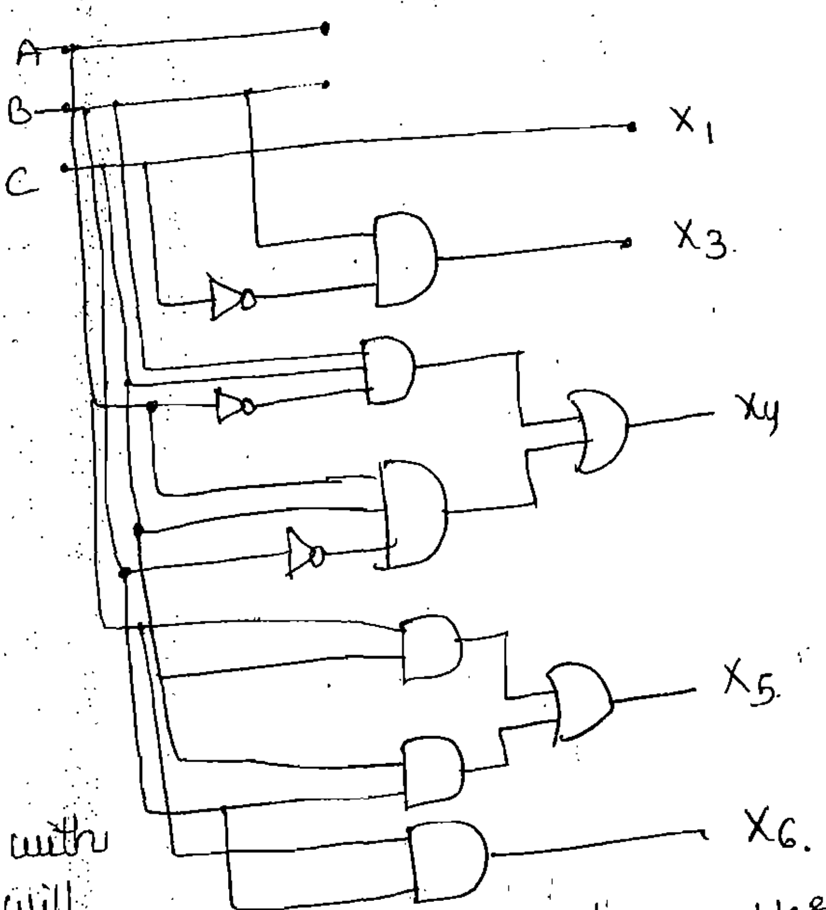



K-map for $X_2 = 0$



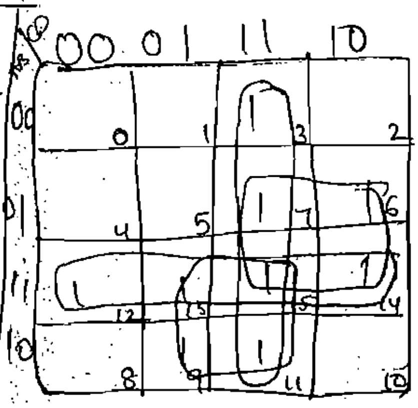

K-map for $X_1 = C$



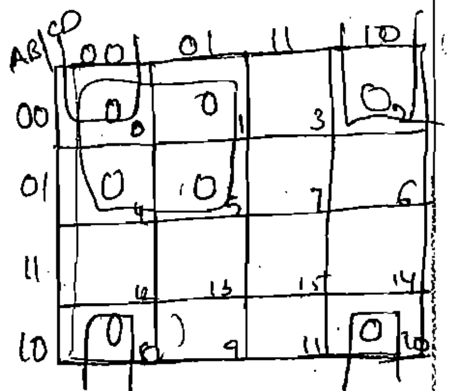




* Design a logic circuit with 4 inputs A, B, C, D that will produce output '1' only whenever two adjacent input variables are 1 is A and D are also to be treated as adjacent implement it using universal logic.

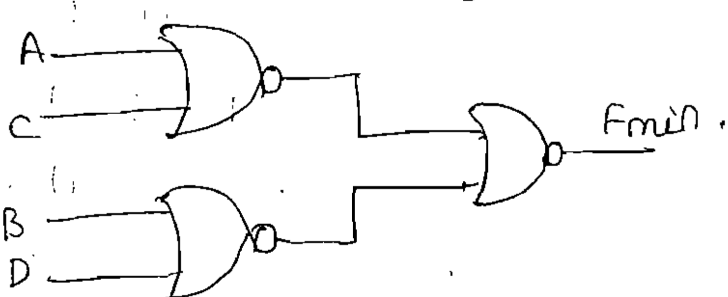
| input A B C D | output f |
|---|---|
| 0 0 0 0 | 0 |
| 0 0 0 1 | 0 |
| 0 0 1 0 | 0 |
| 0 0 1 1 | 1 |
| 0 1 0 0 | 0 |
| 0 1 0 1 | 0 |
| 0 1 1 0 | 1 |
| 0 1 1 1 | 1 |
| 1 0 0 0 | 0 |
| 1 0 0 1 | 1 |
| 1 0 1 0 | 0 |
| 1 1 0 0 | 1 |
| 1 1 0 1 | 1 |
| 1 1 1 0 | 1 |




$F = AB + AD + BC + CD$




$f = (A+C)(B+D)$.




$F_{min} = (A+C)(B+D)$.